

Mint MT Application Note

AN00115-001 - Trapezoidal Move Calculations

Related Applications or Terminology

- **Relative and Absolute Moves**
- **Point to point moves**

Overview

To move motors in a controlled way it is necessary to understand the basic principles behind motion. The motion of moving objects can be specified by a number of parameters, which together define the *Motion Profile*. It is usual to define motion in terms of the following properties: - distance, speed, acceleration, deceleration and move time. This application note describes how to determine the value of any unknown parameters by either calculation or assumption.

We could use some standard equations of motion for uniform acceleration, like:

$$\begin{aligned} v &= u + at \\ s &= ut + \frac{1}{2}at^2 \\ v^2 &= u^2 + 2as \end{aligned}$$

where u = initial speed, v = final speed, a = acceleration and t = time.

But these are difficult to remember and it is not always clear how to apply the equations, however by representing motion in a graphical form and applying some easy to remember and well known formulas we can *Make Motion Simple*.

Scale Factors

A simple example would be a motor that has an encoder fitted that generates 4000 counts per revolution and the axis is to be scaled in degrees, so 1 revolution of the motor creates 4000 encoder counts and moves the output 360 degrees. Therefore the scale factor would be $4000 / 360 = 11.111$ encoder counts per degree. Once scaled all values for distance, speed and acceleration would then be in degrees, degrees per second and degrees per second per second respectively. It is very important to be aware of the units of measurement during all calculations.

A rotary encoder is usually specified in pulses per revolution, because of an effect known as quadrature a motion controller will actually read 4 times this number of encoder counts.

Supported Controllers

NextMovePCI	<input checked="" type="checkbox"/>
NextMoveBX^{II}	<input checked="" type="checkbox"/>
NextMoveST	<input checked="" type="checkbox"/>
NextMoveES	<input checked="" type="checkbox"/>
NextMoveESB	<input checked="" type="checkbox"/>
NextMovee100	<input checked="" type="checkbox"/>
MintDrive^{II}	<input checked="" type="checkbox"/>
Flex+Drive^{II}	<input checked="" type="checkbox"/>

Relevant Keywords

SPEED
ACCEL
DECEL
MOVEA
MOVER

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This is because there are actually two output channels on an encoder (A and B) and each pulse on each channel has a rising and a falling edge. A linear encoder is usually specified in terms of the distance between each count ie 1um (1 micron), this value does not normally need to be multiplied by 4. A 1um linear encoder has 1000 counts per mm so the scale factor would be set to 1000 to scale the axis in mm.

Motion

The distance traveled during a move should be specified in User Units, this distance can be defined in one of two ways, relative or absolute. A *relative* move is defined as the distance to travel from the current position, and an *absolute* move is defined as a move to a specific position relative to some predefined zero position. The distance of travel for an absolute move is the end position - start position. It is important to understand that for the examples in this application note we are referring to the distance traveled, not the position moved to. The MINT keywords MOVER and MOVEA are used to define the end point position as either a relative or an absolute value.

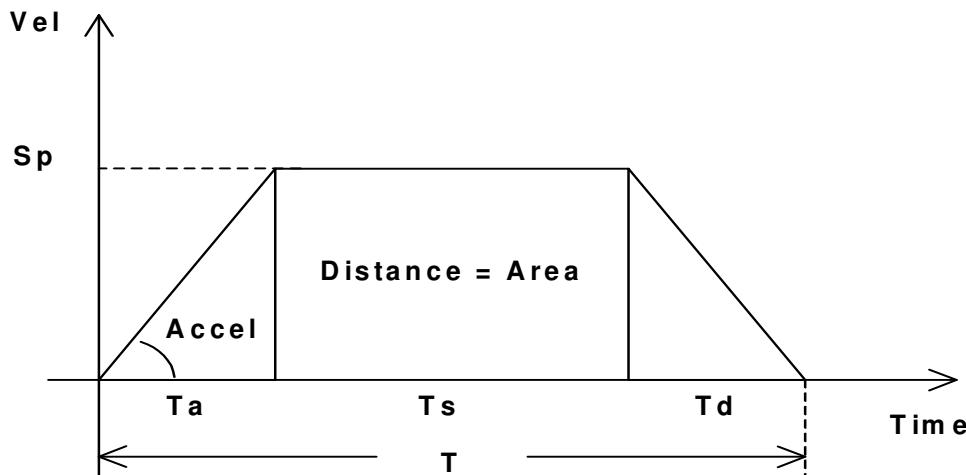
Speed is the velocity attained after the acceleration time and is defined in User Units per second. The MINT keyword SPEED is used to set a value for the slew speed of a move.

Acceleration is the rate of change of speed and is defined in units of user units per second per second. For simple motion we assume a constant rate of acceleration so the acceleration rate can be calculated by dividing the slew speed by the acceleration time. The MINT keywords ACCEL and DECEL are used to set values for acceleration and deceleration rates.

Velocity Time Graphs

To make it easier to understand motion profiles it is usually a good idea to represent them in a graphical format. This can be achieved by plotting a motion property like position or speed against time. The most useful of these graphs is velocity against time.

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The graph shows what is commonly referred to as a trapezoidal move profile, this is because of the shape of the speed profile. There are three sections to the graph, the acceleration part, the constant speed (or slew speed) part and the deceleration part.

From this graph all of the relevant properties can be clearly identified.

- T is the total move time, Ta is the acceleration time, Ts is the slew speed time and Td is the deceleration time.
- Sp is the value of slew speed
- The rate of acceleration is the gradient of the line during the acceleration part of the profile and can be calculated using the formula **Accel = Sp / Ta**
- Similarly the rate of deceleration is defined as the slope of the speed line during the deceleration part of the graph, **Decel = Speed / Td**
- The distance moved is defined as the area under the speed line.

Knowing this information it is possible to use simple calculations to determine any information that is not known to enable us to define a move.

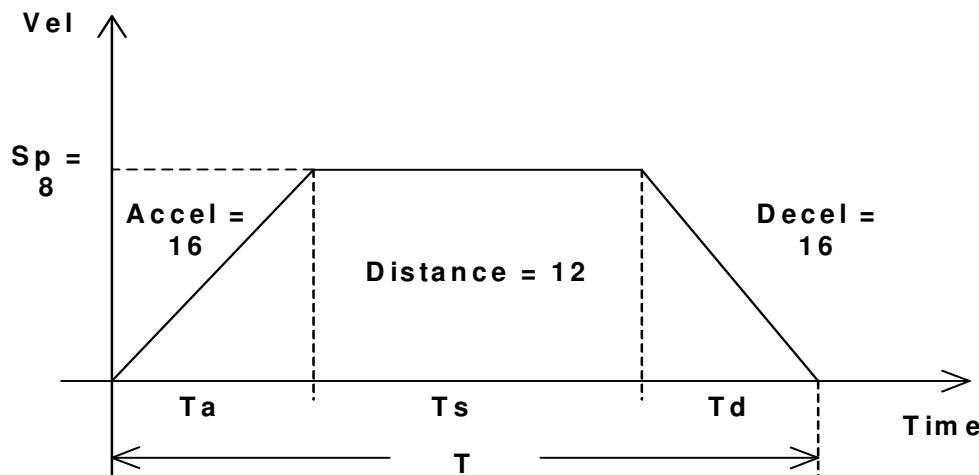
In MINT a move is achieved by setting values for SPEED, ACCEL, DECEL and distance. For example

SPEED = 8
ACCEL = 16

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DECEL = 16
MOVER = 12
GO

This code will create a relative move 12 units long, reaching a speed of 8 units per second, accelerating and decelerating at a rate of 16 units / sec / sec. By putting this information onto the graph we can then determine the other missing information about the move (T, Ta, Ts and Td).



The acceleration rate is given by

$$\begin{aligned}
 \text{Accel} &= \text{Sp} / \text{Ta}, \\
 \text{Ta} &= \text{Sp} / \text{Accel} \\
 &= 8 / 16 \\
 &= 1/2 \text{ sec}
 \end{aligned}$$

Similarly $\text{Td} = \text{Sp} / \text{Decel} = 1/2 \text{ sec}$

Looking at the shape of the acceleration part, it is a triangle. We know that the distance traveled is equal to the area under the speed curve so the distance traveled during acceleration can be calculated given by using the equation for the area of a triangle =

$$\text{Area} = 1/2 \times \text{base} \times \text{height}$$

Therefore

$$\text{Da} = 1/2 \times \text{Ta} \times \text{Sp}$$

Therefore the distance traveled during acceleration is

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$$Da = \frac{1}{2} \times \frac{1}{2} \times 8 = 2 \text{ units.}$$

As the deceleration rate is the same as the acceleration then the distance traveled during deceleration is the same as that for the acceleration part, 2 units.

We know that the total distance traveled is the sum of the other distances and from this we can calculate the distance traveled at Slew Speed

$$\begin{aligned} D &= Da + Ds + Dd \\ \text{so } Ds &= D - Da - Dd \\ &= 8 \text{ units.} \end{aligned}$$

The area under the slew speed section is a square, so

$$\begin{aligned} Ds &= Sp \times Ts \\ \text{Rearranged gives } Ts &= Ds / Sp \\ &= 8 / 8 \\ &= 1 \text{ sec} \end{aligned}$$

$$\begin{aligned} \text{so the total move time } T &= Ta + Ts + Td \\ &= \frac{1}{2} + 1 + \frac{1}{2} \\ &= 2 \text{ seconds} \end{aligned}$$

It is quite common that not all of the information that is required to define the move is provided. For example, perform a move of 20 units in 600ms. In these situations it is worth adding the known information to the speed vs time graph and then making some reasonable assumptions.

In this example the slew speed and the acceleration / deceleration rates have not been defined. It might not be immediately clear what values we should use for these but by looking at the graph we can easily assume values for the Accel and Decel time.

With a total move time of 600ms, we could use $Ta = 200\text{ms}$, $Ts = 200\text{ms}$ and $Td = 200\text{ms}$. We are free to use any values we like but choosing values that make the maths easier is always a good point to start. By setting the Accel and Decel times to be the same value usually simplifies the calculations.

So using the information we have been given together with the assumptions we have made enables us to calculate the missing information.

Consider the shape of the velocity time graph, to find the missing information we need to know the equation for the area of this trapezoidal shape.

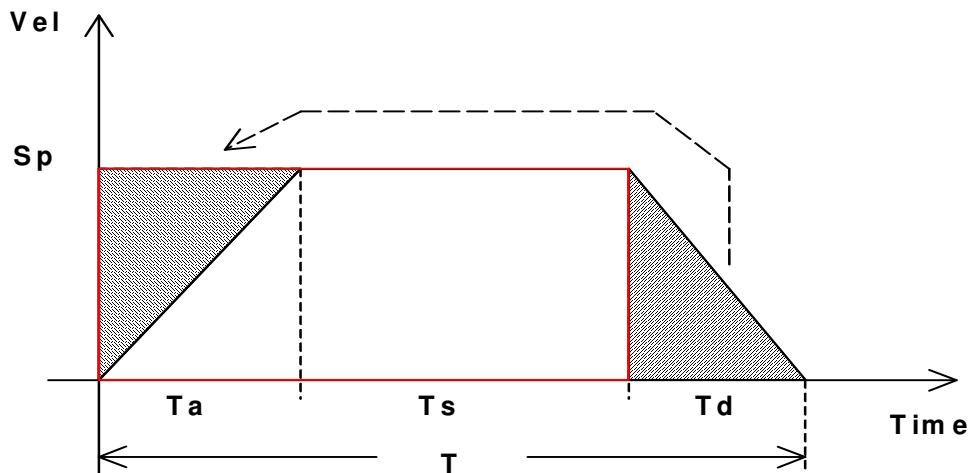
$$A = \frac{1}{2} (a + b) \times c$$

where A is the area or distance traveled, a is the Slew speed time (Ts) and b is the total move time T.

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$$\text{Therefore } \begin{aligned} \text{Distance} &= \frac{1}{2} (\text{Ts} + \text{T}) \times \text{Sp} \\ \text{Sp} &= 2 \times \text{distance} / (\text{Ts} + \text{T}) \end{aligned}$$

This is a simple enough formula that can be determined by looking at the graph but it does require that you remember the formula for the area of a trapezoid. To make it even easier if we look at the graph we can see the individual shapes that make up the trapezoid, two triangles and a rectangle. If the Accel time is the same as the Decel time then the area of the Decel triangle is the same as the area above the Accel triangle. By imagining that this area is cut out, turned upside down and inserted above the Accel part the resultant shape becomes a rectangle.



Where the area of the red rectangle is the same as the original trapezoidal shape.

This method only requires that we remember the formula to calculate the area of a rectangle ($A = a \times b$) or using motion parameters **Distance = Sp x (Ta + Ts)**

Rearranged this give us

Once we know the value of Speed and Accel time then

(If Accel time and Decel time are the same then Decel rate will be the same as Accel rate)

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In the previous example a 20 unit move must be completed in 600ms, we do not know all values for SPEED, ACCEL or DECEL but we have assumed Accel time, slew speed time and Decel time to all be 200ms.

Using equation (1)

$$\begin{aligned}Sp &= \text{Distance} / (\text{Ta} + \text{Ts}) \\&= 12 / (0.2 + 0.2) \\&= 30 \text{ units / sec}\end{aligned}$$

and using equation (2)

$$\begin{aligned}\text{Accel} &= \text{Sp} / \text{Ta} \\&= 30 / 0.2 = 150 \text{ units / s / s}\end{aligned}$$

(Remember that all units must be scaled units or seconds)

So the code in the MINT program needs to be

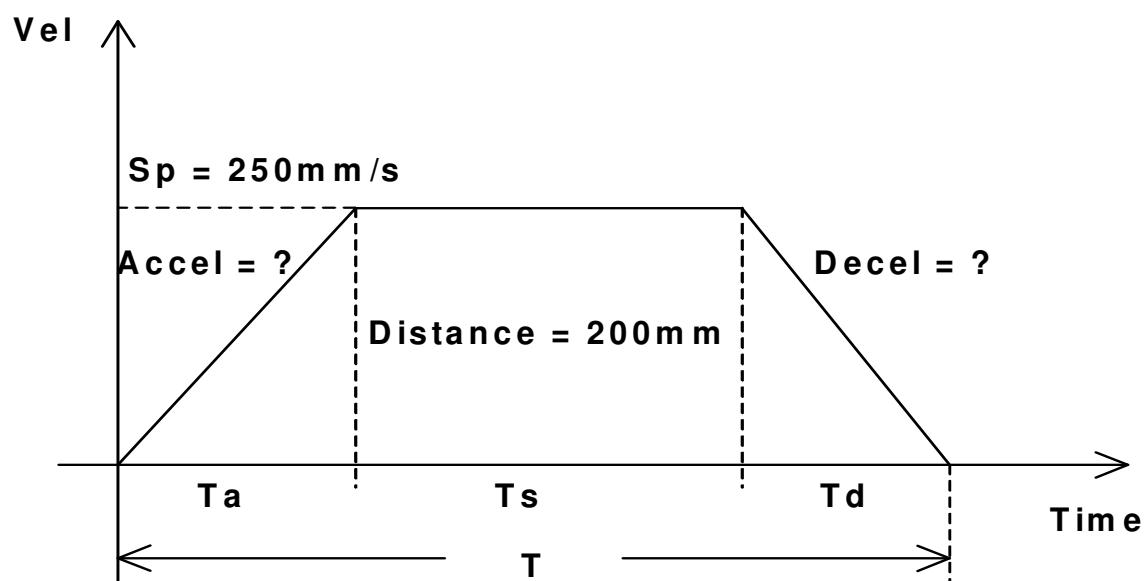
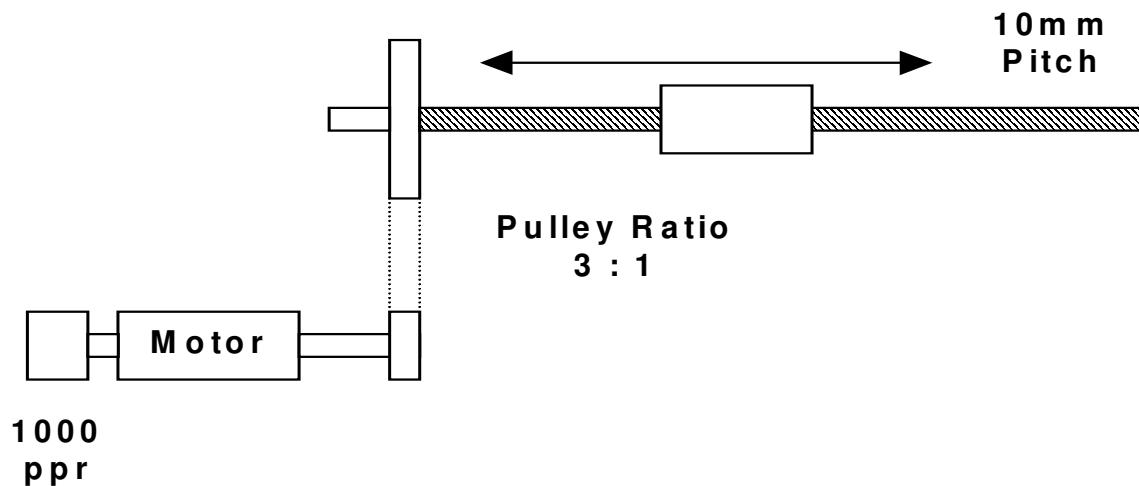
```
SPEED = 30
ACCEL = 150
DECCEL = 150
MOVER = 20
GO
```

This process shows that if motion can be represented on a velocity time graph and in the form of simple shapes it is straight forward to manipulate the information available, make reasonable assumptions and calculate any missing values. The only formula that need to be remembered are equations for the areas of triangles and rectangles and how to calculate the gradient of a line.

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Exercise

Write a MINT program to perform a 1 second move of 200mm, reaching a speed of 250mm/s. The motor has a 1000 pulse per rev encoder driving a 10mm pitch ball-screw connected via a 3:1 pulley ratio.



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Exercise Answer

First calculate the scale factor

1 rev of motor gives 4000 counts

1 rev of motor gives $1/3 * 10\text{mm}$ of motion on the output of the ball-screw.

$$\text{Therefore SCALE} = \frac{4000}{1/3 \times 10} = 1200 \text{ encoder counts per mm}$$

There is not enough information to determine the accel and decel rates so it is necessary to make some assumptions. Assume an acceleration time of 200ms, a slew speed time of 600ms and a deceleration time of 200ms.

The Acceleration rate is therefore $250 / 0.2 = 1250 \text{ mm/s/s}$, which is the same as the deceleration rate.

Therefore the code would be

```
SCALE = 1200
SPEED = 250
ACCEL = 1250
DECEL = 1250
MOVER = 200
GO
```