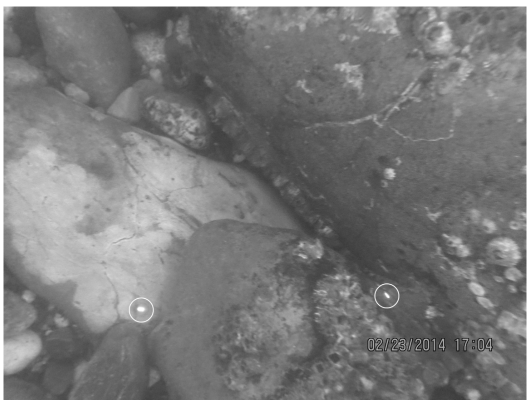
26 February 2015 Homework Name:______________________________ ⑥

Due Thursday 5 March 2015 Date: ___/___/___ Day No.: _______

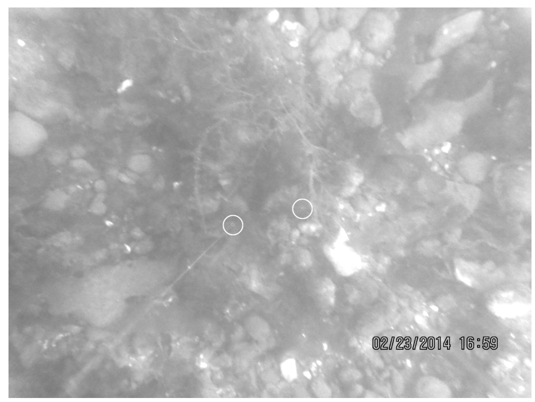
Underwater Field of View and Depth

Introduction

The camera used with your ASVs will not only take pictures, but also determine the depth of the water by the use of two parallel laser beams. Where the laser beams touch the bottom, they will (hopefully) show up as two very bright dots in the resulting image. In order to understand how this depth-determination process works, you need to grasp some "optics math." All of this has to do with the camera's field of view compared to the apparent spacing of the two laser dots. The smaller the distance between the two laser dots in the image, the farther the bottom will be from the camera. Here are a couple sample images that illustrate this relationship:



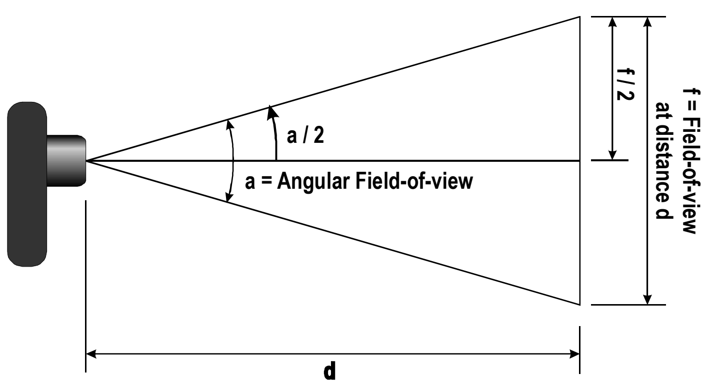
Bottom Near: Dots Far Apart



Bottom Far: Dots Close Together

What is "Field of View?"

When you take a photo with any camera, the image that results is confined to what the camera "sees" at the subject distance. The width and height of this confined rectangle are what's known as the **field of view** at the subject distance. Obviously, the farther the subject is from the camera, the bigger the "frame" around that subject becomes. Here, we are concerned only with the horizontal field of view, as the following diagram illustrates:



From trigonometry, you get the following relation:

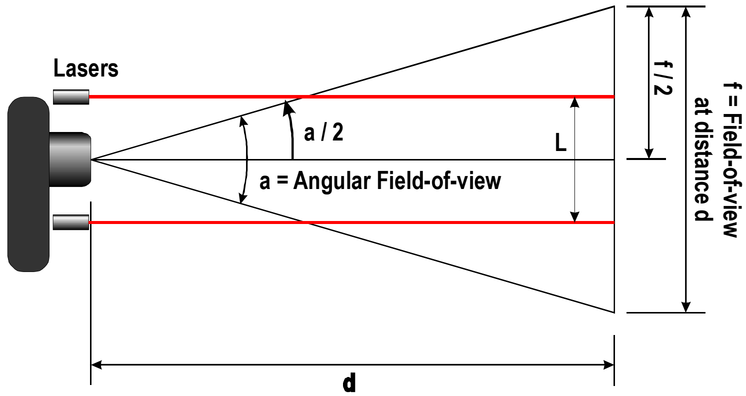
$$(f/2) / d = \tan(a/2)$$

Solving for f, you get:

$$f = 2d \tan(a/2)$$

Using Lasers to Determine Distance

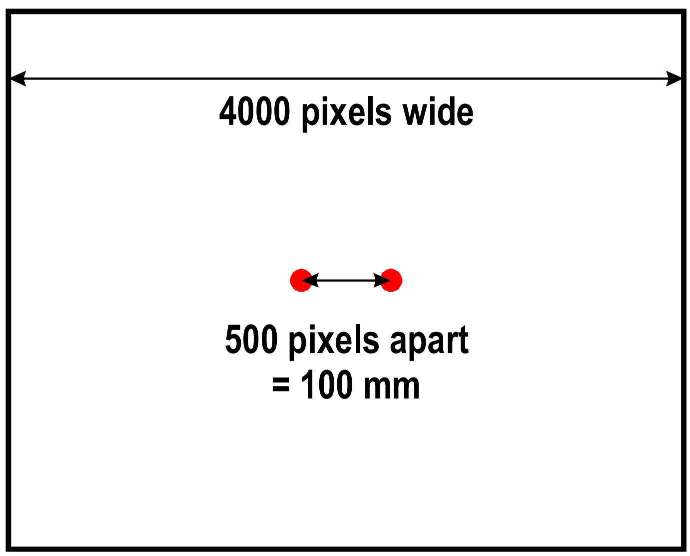
By introducing the lasers, you get a situation that looks like this:



Because the laser beams are parallel, the distance **L** never changes. So you always know the actual spacing between dots in the photo. At a fixed zoom level, you also know the angle **a**. If you don't, you can calibrate it by photographing a horizontal meter stick from a short distance **d** to obtain **f**, the portion of the meter stick visible in the image. Then you can compute **a** as

$$a = 2 \tan^{-1}(f / 2d)$$

Since **L** is a given, and if you know **a**, you can compute **f** from any photo that shows the two laser dots. For example, if the lasers are 100mm apart, and if the laser dots are 500 pixels apart in an image that's 4000 pixels wide,



Field of view = (4000 / 500) • 100mm

you know that

$$f / 100mm = 4000 / 500, \text{ or}$$

$$f = (4000 / 500) \bullet 100mm = 800mm$$

In general,

$$f = photo\_pixel\_width \bullet L / pixels\_between\_laser\_dots$$

Once you know **f**, **d** can be computed from

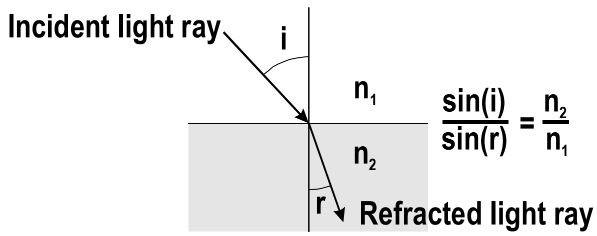
$$(f/2) / d = \mathbf{tan}(a/2), \text{ or}$$

$$d = f / (2\ \mathbf{tan}(a/2))$$

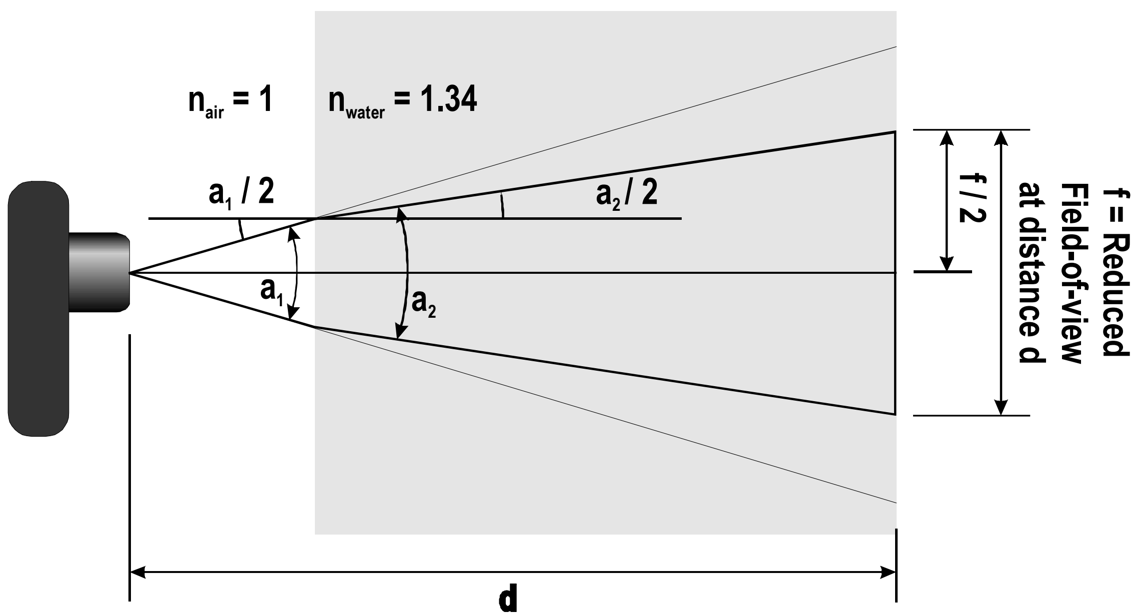
Just Add Water and Stir (Things Up)

At a known zoom level, a camera's field of view in water will be different from its field of view in air. This is due to **refraction**. Light entering a tranparent medium like water from a dissimilar medium like air will be deflected at an angle. Every transparent medium has a characteristic **index of refraction**, **n**. For air, **n** is 1.0. For seawater, **n** is approximately 1.34. That number is approximate because **n** varies a little with temperature, wavelength of light, and salinity. For our purposes, though, 1.34 will be good enough.

According to **Snell's Law**, the amount the light is deflected at the boundary between two media is related to the angle of incidence and the ratio of the two indices of refraction. Here's an illustration, along with the formula:



When the camera is pointed into the water, this reduces the camera's angular field of view as shown below:



From Snell's Law, we get:

$\sin(a_1 / 2) / \sin(a_2 / 2) = 1.34 / 1 = 1.34$

$\sin(a_2 / 2) = \sin(a_1 / 2) / 1.34$

$a_2 / 2 = \sin^{-1}(\sin(a_1 / 2) / 1.34)$

$a_2 = 2 \sin^{-1}(\sin(a_1 / 2) / 1.34)$

If the camera's lens is very close to the water, as it will be with the ASVs, we can ignore the slight initial spread in the field of view caused by $\mathbf{a_1}$ above, and just compute **d** based on **f** and $\mathbf{a_2}$. So, in the above illustration with the lasers, all you have to do is substitute $\mathbf{a_2}$ for **a**, and everything works out again. **Note:** Because the laser beams are perpendicular to the water, they are not affected by refraction.

Now you know everything you need to complete the exercise on the back of this page!

Exercise

You calibrate the camera's field of view by photographing a horizontal meter stick at a distance of 0.5 meters. When you look at the image, you see only 50% of the meter stick's length. From this you can compute $\mathbf{a_1}$, the camera's angular field of view in air, which is

$\mathbf{a_1}$ = __________ degrees

From $\mathbf{a_1}$, you can compute $\mathbf{a_2}$, the camera's angular field of view in seawater, which is

$\mathbf{a_2}$ =___________ degrees

You have laser beams spaced 100mm apart, and you take a picture of the bottom. When you look at the image, which is 4000 pixels wide, you measure the distance between the two laser dots as 250 pixels. How deep was the water when you took the picture?

d = ____________ meters