Application note
Trapezoidal move calculations

The motion of an axis moving from one point to another can be specified by a number of parameters, which together define the motion profile. For a simple trapezoidal profile these parameters are move distance, speed, acceleration and deceleration

Introduction

We could use some standard equations of motion for uniform acceleration, like:

$$v = u + at$$
$$s = ut + 1/2at^2$$
$$v^2 = u^2 + 2as$$

where u = initial speed, v = final speed, a = acceleration and t = time.

But these are difficult to remember and it is not always clear how to apply the equations, however by representing motion in a graphical form and applying some easy to remember and well known formulas we can *Make Motion Simple*.

Scale Factors

A Mint scale factor defines how many encoder counts occur in travel of one user unit. A simple example would be a motor that has an encoder fitted that generates 4000 counts per revolution and the axis is to be scaled in degrees, so 1 revolution of the motor creates 4000 encoder counts and moves the output 360 degrees. Therefore the scale factor for a user unit of "degrees" would be 4000 / 360 = 11.111 encoder counts per degree. Once scaled all values for distance, speed and acceleration would then be in degrees, degrees per second and degrees per second per second respectively. It is very important to be aware of the units of measurement during all calculations.

A rotary encoder is usually specified in pulses per revolution, because of an effect known as quadrature a motion controller will actually read 4 times this number of encoder counts. This is because there are actually two output channels on an encoder (A and B) and each pulse on each channel has a rising and a falling edge. A linear encoder is usually specified in terms of the distance between each count i.e. 1µm (1 micron), this value does not normally need to be multiplied by 4. A 1µm linear encoder has 1000 counts per mm so the scale factor would be set to 1000 to scale the axis in mm of linear travel.

Motion

The distance traveled during a move should be specified in User Units, this distance can be defined in one of two ways, relative or absolute. A *relative* move is defined as the distance to travel from the current position, and an *absolute* move is defined as a move to a specific position relative to some predefined zero position. The distance of travel for an absolute move is the end position - start position. It is important to understand that for the examples in this application note we are referring to the distance traveled, not the
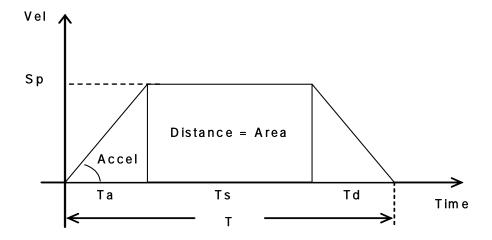
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position moved to. The MINT keywords MOVER and MOVEA are used to define the end point position as either a relative or an absolute value respectively.

Speed is the velocity attained after the acceleration time and is defined in User Units per second. The MINT keyword SPEED is used to set a value for the slew speed of a move.

Acceleration is the rate of change of speed and is defined in units of user units per second per second. For simple motion we assume a constant rate of acceleration so the acceleration rate can be calculated by dividing the slew speed by the acceleration time. The MINT keywords ACCEL and DECEL are used to set values for acceleration and deceleration rates. It is also possible to define acceleration and deceleration times using the ACCELTIME and DECELTIME keywords if required.

Velocity Time Graphs

To make it easier to understand motion profiles it is usually a good idea to represent them in a graphical format. This can be achieved by plotting a motion property like position or speed against time. The most useful of these graphs is velocity against time.

The graph shows what is commonly referred to as a trapezoidal move profile, this is because of the shape of the speed profile. There are three sections to the graph, the acceleration part, the constant speed (or slew speed) part and the deceleration part.

From this graph all of the relevant properties can be clearly identified.
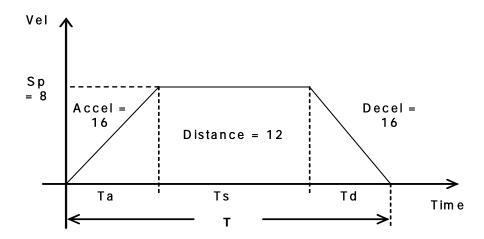
- T is the total move time, Ta is the acceleration time, Ts is the slew speed time and Td is the deceleration time.
- Sp is the value of slew speed
- The rate of acceleration is the gradient of the line during the acceleration part of the profile and can be calculated using the formula **Accel = Sp / Ta**
- Similarly the rate of deceleration is defined as the slope of the speed line during the deceleration part of the graph, **Decel = Speed / Td**
- The distance moved is defined as the area under the speed line.

Knowing this information it is possible to use simple calculations to determine any information that is not known to enable us to define a move.

In MINT a move is achieved by setting values for SPEED, ACCEL, DECEL and distance. For example

```
SPEED = 8
ACCEL = 16
DECEL = 16
MOVER = 12
GO
```

This code will create a relative move 12 units long, reaching a speed of 8 units per second, accelerating and decelerating at a rate of 16 units / sec / sec. By putting this information onto the graph we can then determine the other missing information about the move (T, Ta, Ts and Td).

The acceleration rate is given by:

Accel = Sp / Ta,
   Ta = Sp / Accel
       = 8 / 16
       = ½ sec

Similarly **Td = Sp / Decel = ½ sec**

Looking at the shape of the acceleration part, it is a triangle. We know that the distance traveled is equal to the area under the speed curve so the distance traveled during acceleration can be calculated given by using the equation for the area of a triangle:

    Area = ½ x base x height

Therefore   **Da = ½ x Ta x Sp**

Therefore the distance traveled during acceleration is…

        Da = ½ x ½ x 8 = 2 units.

As the deceleration rate is the same as the acceleration then the distance traveled during deceleration is the same as that for the acceleration part, 2 units.

We know that the total distance traveled is the sum of the other distances and from this we can calculate the distance traveled at Slew Speed

           D = Da + Ds + Dd
=>   **Ds = D - Da - Dd**
        Ds = 8 units.

The area under the slew speed section is a square, so…

        Ds = Sp x Ts

Rearranged gives   **Ts = Ds / Sp = 8 / 8 = 1 sec**

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Therefore the total move time…

> T = Ta + Ts + Td
> = ½ + 1 + ½
> = 2 seconds

It is quite common that not all of the information that is required to define the move is provided. For example, perform a move of 20 units in 600ms. In these situations it is worth adding the known information to the speed vs time graph and then making some reasonable assumptions.

In this example the slew speed and the acceleration / deceleration rates have not been defined. It might not be immediately clear what values we should use for these but by looking at the graph we can easily assume values for the Accel and Decel time.

With a total move time of 600ms, we could use Ta = 200ms, Ts = 200ms and Td = 200ms (i.e. split the total move into three equally timed segments….this is known as the "third rule"). We may also consider using Ta = 150ms, Ts = 300ms and Td = 150ms (i.e. use one quarter of the available time for both the acceleration and deceleration times…..this is known as the "quarter rule").
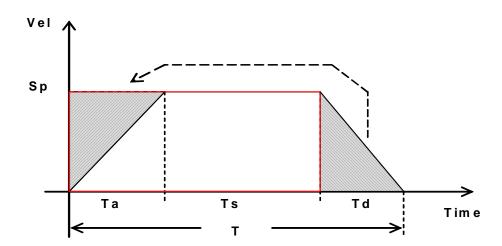
We are free to use any values we like but choosing values that make the maths easier is always a good point to start. By setting the Accel and Decel times to be the same value usually simplifies the calculations. Using the information we have been given together with the assumptions we have made enables us to calculate the missing information.

Consider the general shape of the velocity time graph, to find any missing information (such as the slew speed for our move) we can use the general equation for the area of this trapezoidal shape…

> $A = ½ (a + b) . c$

…where A is the area (or distance traveled), a is the Slew speed time (Ts) , b is the total move time T and c is the speed (Sp)

=>   Distance = ½ (Ts + T) x Sp
        Sp = 2 x distance / (Ts + T)

This is a simple enough formula that can be determined by looking at the graph but it does require that you remember the formula for the area of a trapezoid. To make it even easier if we look at the graph we can see the individual shapes that make up the trapezoid, two triangles and a rectangle. If the Accel time is the same as the Decel time then the area of the Decel triangle is the same as the area above the Accel triangle. By imagining that this area is cut out, turned upside down and inserted above the Accel part the resultant shape becomes a rectangle…

…where the area of the red rectangle is the same as the original trapezoidal shape.

This method only requires that we remember the formula to calculate the area of a rectangle ($A = a . b$) or expressed in terms of motion parameters…

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Distance = Sp x (Ta + Ts)

Rearranged this give us

Sp = Distance / (Ta + Ts) ………………(1)

Once we know the value of Speed and Accel time then

Accel = Sp / Ta…………………..............(2)

(If Accel time and Decel time are the same then Decel rate will be the same as Accel rate)

In the previous example a 20 unit move must be completed in 600ms, we do not know a values for SPEED, ACCEL or DECEL but we have assumed Accel time, slew speed time and Decel time to all be 200ms using the third rule.

Using equation (1)   Sp = Distance / (Ta + Ts)
                                        = 20 / (0.2 + 0.2)
                                        = 50 units / sec

Using equation (2)   Accel = Sp / Ta
                                          = 50 / 0.2 = 250 units / s / s

(Remember that all units must be scaled into user units or seconds)

So the code in the MINT program to make this 20 unit move on axis 0 in 600ms would be…

```
SPEED(0) = 50
ACCEL(0)  = 250
DECEL(0)  = 250
MOVER(0)  = 20
GO(0)
```

Using the keywords ACCELTIME and DECELTIME instead we could remove the need to calculate the ACCEL and DECEL rates and use our times directly…

```
SPEED(0) = 50
ACCELTIME(0)  = 200
DECELTIME(0)  = 200
MOVER(0)  = 20
GO(0)
```
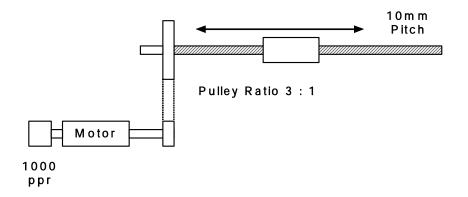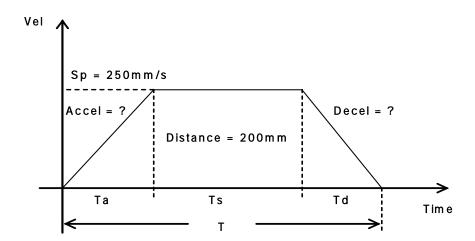
(note that the units for ACCELTIME and DECELTIME are ms).

This process shows that if motion can be represented on a velocity time graph and in the form of simple shapes it is straight forward to manipulate the information available, make reasonable assumptions and calculate any missing values. The only formula that need to be remembered are equations for the areas of triangles and rectangles and how to calculate the gradient of a line.

Exercise

Write a MINT program to perform a move of 200mm on a ball-screw in 1 second. The motor has a 1000 pulse per rev encoder driving a 10mm pitch ball-screw connected via a 3:1 pulley ratio.

Exercise Answer

First calculate the scale factor
1 rev of motor gives 4000 counts
1 rev of motor gives 1/3 * 10mm of motion on the output of the ball-screw.

Therefore SCALE = $\dfrac{4000}{(1/3) \times 10}$ = 1200 encoder counts per mm

There is not enough information to determine the accel and decel rates so it is necessary to make some assumptions. Assume an acceleration time of 200ms, a slew speed time of 600ms and a deceleration time of 200ms.

As the accel and decel times are the same we can use the formula…

Speed = Distance / (Ta + Ts)

….to calculate the slew speed….

Speed = 200 / (0.2 + 0.6) = 250 mm/sec

The Acceleration rate is therefore 250 / 0.2 = 1250 mm/s/s, which is the same as the deceleration rate.

Therefore the code would be…

```
SCALE(0) = 1200
SPEED(0) = 250
ACCEL(0) = 1250
DECEL(0) = 1250
MOVER(0) = 200
GO(0)
```

…or using ACCELTIME and DECELTIME…

```
SCALE(0) = 1200
SPEED(0) = 250
ACCELTIME(0) = 200
DECELTIME(0) = 200
MOVER(0) = 200
GO(0)
```

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