Fuzzy Linguistic Controllers Applied to Decouple Control Loop Effects

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***Abstract* — A fuzzy linguistic controller has been developed and implemented with the aim to cope with interactions between control loops due to coupling effects. To access the performance of the proposed approach several experiments have also been conducted using the classical PID controllers in the control loops. A mixing process has been used as test bed of all controllers experimented and the corresponding dynamic model has been derived. The successful results achieved with the fuzzy linguistic controllers suggests that they can be an alternative to classical controllers when in the presence of process plants where automatic control as to cope with coupling effects between control loops.**

Keywords-fuzzy logic; fuzzy sets; fuzzy linguistic controllers; feedback control loop; inference mechanism; PID controllers.

I. INTRODUCTION

Due to the enormous influence that the mechanistic paradigm or "classic" still plays today in the scientific landscape, the conventional mathematical techniques were, and still continue to be applied in the analysis of humanistic systems. Given these facts, is becoming increasingly clear that the complexity of specific problems and the particular characteristics that these systems offer, requiring approaches that should be significantly different, both in spirit and in substance from traditional methods. One of these approaches that in the last four decades has been increasingly recognized as a provider of solutions for many of these problems is the so-called Theory of Fuzzy Sets [1].

The theory appeared explicitly in sixties. However, the French expression "ensemble flou" has been used by Menger in 1951, when based on a probabilistic interpretation, launched what is now called the operation, *max-product*, of a "fuzzy" transitive relationship.

Regarding the comparison between exact sciences and the humanities, much of the potential conflicts between them, were overcome by the extension of “Fuzzy Set Theory” to a new field of research called "approximate reasoning". Do not go into details about their principles. For now we will concern ourselves only with the fact that this approach is based on two main concepts as follows:

a) The linguistic variable, which is a variable whose values are words or sentences in natural language (e.g., "age" can be seen as a linguistic variable if its values are considered "young", "old", etc.);
b) The rules of inference and "fuzzy" operations, which enable us to reach a conclusion from premises that are neither accurate nor too many inaccurate.

Fuzzy Sets and Fuzzy Logic cover not only these issues but also the areas where computer simulations of interactions between humans, between humans and machines or between machines, are in fact involved. One of these areas where complexity, uncertainty, linguistic variables and linguistic rules as well as the interaction man/machine are all related has to do with the theory of so-called Linguistic Controllers. This is an area of knowledge intrinsic of control engineering based on fuzzy concepts, which nowadays knows more and more applications in products of mass consumption until the search for solutions to particular problems in manufacturing industry.

The first practical application of the Fuzzy Sets Theory to control problems was carried out by Mamdami and Assilian [2]. In this classic article (and now historical), entitled "An Experiment in Linguistic Synthesis with Fuzzy Logic Controller" they show as the main theoretical fuzzy operations can be used efficiently and effectively in modelling and control systems, which at the time were unable to be adequately treated using conventional techniques. Since then, the revolutionary Mandami’s approach has resulted in thousands of scientific papers concerned with fuzzy control, applied to several areas different each other [3][4][5].

In this paper a fuzzy linguistic controller has been developed and implemented to cope with interactions between feedback control loops due to coupling effects. A mixing process has been used as test bed of the proposed approach. Furthermore, in order to access the fuzzy controller performance a results comparison is made by using the classical PID controllers in the control loops.

The paper has a structure as follows. After this introductory section comes the section II where is made a brief introduction to the fuzzy sets theory allowing the reader to understand how fuzzy controllers are design and work. The following section describes the mixing process used as test bed of the proposed fuzzy control approach. Section IV describes the controllers implemented during the current studies and provides a results analysis and discussion. The last section draws some concluding remarks and outline future research work.

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II. FUZZY LINGUISTIC CONTROLLERS

According to the assumptions of Zadeh, there are many sets for which are not possible to establish precise criteria to define the elements that make up a particular set [1]. To solve this type of situations, Zadeh suggests establishing a membership degree, which will be a degree of belonging to a specific set, so that it ensures a gradual transition, in contrast to the abrupt transition from classical set theory, between being an element of a set and not being an element of the same set [1]. Thus, a fuzzy set is characterized by the membership values of all its elements, where the membership value of a particular element is usually a real number between 0 and 1 and is often represented by the Greek letter μ. The higher this number is, the greater the degree to which certain element belongs to the set in question.

The membership function of a fuzzy set, although similar, is not the same thing as a distribution function of a statistical probability. Each element in the universe is associated with a membership degree to a particular fuzzy set, which may eventually be zero. The set of all elements with non-zero membership degree to a particular fuzzy set constitutes what is known as the fuzzy set support. The function that assigns the membership degree of each element, x, of the universe, is called the membership function, μ *(x)*. From computers point of view, there are two ways to make the implementation of membership functions: continuous and discrete ways. In the continuous form, the membership function representation is performed using a certain mathematical function. The implementation of the membership functions in discrete mode as well, the universe in which they are located, is made via a discrete set of points, commonly referred to as a vector. Either of these forms for the implementation of the membership functions is valid. However, in general, the continuous representation requires more computing time, while the discrete representation requires more memory space.

In a formal way, the fuzzy set A is represented by a collection of ordered pairs, in the following form, $A = \{(x, \mu(x))\}$. In that, x, belongs to the universe and $\mu(x)$ is the membership degree of x to A. A single pair $(x, \mu (x))$ is called a fuzzy singleton; this way, a fuzzy set can be seen as the union of all singletons that constitute it.

In a similar way to what happens with the algebraic variables that take numbers as values, linguistic fuzzy variables take words or phrases as values [6]. The set of values that can be taken by a particular linguistic fuzzy variable is called the set of terms. Each value in the set of terms is a fuzzy variable defined according to the base variable. In turn, the base variable defines the universe of discourse for all fuzzy variables.

In order to formally define the basic operations with fuzzy sets, consider a and b, defined as fuzzy sets in a universe of discourse:

a) According to the fuzzy sets theory, the intersection of a with b is defined as follows: $a \; and \; b \triangleq min \, (a, b)$. The operation, min, is an operation performed element by element by determining the minimum value of the membership degree of the elements that belong to both sets, a and b.

b) Similarly, the union of, a with b, is defined according to the following relationship: $a \; or \; b \triangleq \max \; a, b)$. The operation, max is also an operation performed element by element, determining the maximum membership degree of elements belonging to both sets, a and b.

c) The complement of, a, is defined as follows: $not \; a \triangleq 1 - a$. Where the membership degree of each element of, a, is subtracted from 1.

However, there are other possible definitions for the basic operations, but the definitions using the operators *max* and *min* are the most common.

Furthermore, in any fuzzy system the relationships between objects are characterized by having a key role. Thus, we can have relationships between elements belonging to the same fuzzy set, for example: a measure is greater than other; certain event occurred earlier than the other, etc. We also have relationships between different fuzzy sets, for example: a speed measure is big and its variation is positive; the x coordinate is big and y coordinate is small, etc.

Formally, a binary fuzzy relationship, R, of a fuzzy set, a, for a fuzzy set, b, is a subset of the Cartesian product of U with V, these being, respectively, the universes of a and b. The Cartesian product, according to the classical set theory, is the set of all possible elements combinations of U and V.

The general rule, when we combine or compose fuzzy relationships, is to take the lesser membership degree in the "series links" and the highest membership degree in the "parallel links". Formally, the definition of composition is as follows: given two binary relationships, R and S, each represented by a matrix, such that the number of columns of R is equal to the number of lines of S, then its composition is given by, $R \circ S$, where, $\circ$, represents the inner product, *OR-AND*, of matrices. The inner product of matrices, *OR-AND*, is similar to the ordinary matrices product, except that the multiplication is replaced by the logic operation, *AND*, and the sum is replaced by the logical operation, *OR*. So, assuming that R is a matrix, $m \times p$, and that S is a matrix, $p \times n$, then the result of the composition of R with S is a matrix, T, in which its elements, t_{ij}, are obtained by combining the line, i of R, with the column, j of S, such that,

$$t_{ij} = (r_{i1} \wedge s_{1j}) \vee (r_{i2} \wedge s_{2j}) \vee \dots \vee (r_{ip} \wedge s_{pj}) = \bigvee_{k=1}^{p} r_{ik} \wedge s_{kj} \qquad (1)$$

In the equation above, the logical operations *AND* and *OR*, were replaced by symbols, □ and □, as a shorthand notation. Using these definitions, the composition is no more than what is known in the literature by, *max-min* composition [6]. Note also that if R is a relationship, from a to b, and S is a relationship, from b to c, then the composition of R with S, is a relationship, from a to c (Transitive property).

Thus, a control fuzzy heuristic rule can be written as "*if e* is positive big *then u* is positive big" being mathematically translated by the implication operation, because the value of *e* (error signal in a feedback control loop) imply the value of *u* (control signal in a feedback control loop). Formally, the definition of implication is as follows [7][8]: being, *a* and *b*, two fuzzy sets, not necessarily defined in the same universe of discourse, the implication, *a* implies *b*, is defined as follows, $a => b \triangleq a \; x \; b$. Where *x* represents the outer product using the logic operation, *AND*. Thus, the outer product applies the logic operation, *AND*, to each element of the Cartesian product of two arguments. Then, considering that the vector, *a*, can be represented by a column vector and, *b*, can be represented by a row vector, their outer product can be obtained through the following relationship,

$$\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} x [b_1 \quad b_2 \quad \dots \quad b_n] =$$

$$= \begin{bmatrix} a_1 \wedge b_1 & a_1 \wedge b_2 & \dots & a_1 \wedge b_n \\ a_2 \wedge b_1 & a_2 \wedge b_2 & \dots & a_2 \wedge b_n \\ \vdots & \vdots & \vdots & \vdots \\ a_n \wedge b_1 & a_n \wedge b_2 & \dots & a_n \wedge b_n \end{bmatrix} \quad (2)$$

To draw conclusions from a knowledge base is necessary to use a mechanism capable of producing an output from a set of heuristic rules such as "*if-then*". This is achieved through the inference mechanism [9]. The verb "infer" means to conclude from given evidence, work out, or do specific things as a logical consequence of something. To understand this concept consider the function, $y = f(x)$, where *f* is any function, *x* is the independent variable and *y* is the result. The value of *y* is inferred from $f(x)$, giving a specific value to *x*. Another analogy might be an electrical circuit: given the circuit topology and all its components, we can infer all values of current intensity and voltage in the circuit from the current intensity and voltages in the sources.

In the context of Boolean Logic there is a famous rule of inference known as modus ponens, which says: If it is known that *a* => *b* is true and that, *a* is also true, then we can infer that *b* is true. The Fuzzy Logic generalizes this inference rule to what is known as generalized modus ponens (GMP). Note that the fuzzy logic will reach the same conclusions in the presence of fuzzy sets, a' and b', a slightly different from fuzzy sets, *a* and *b*. The inference rule, generalized modus ponens, is based on compositional rule of inference being particularly important in the implementation of applications based on fuzzy logic. Hence, the compositional rule of inference has the following definition: being, *R*, a relation of the universe, u_1, to the universe, u_2, and, *a*, a fuzzy set defined in u_1, then, $a \circ R = b$, where, *b*, is a fuzzy set defined in u_2, induced by *a*, and, °, is the composition operation.

A knowledge base of a system designed based on fuzzy logic, is obviously made up of several heuristic rules, as generally happens in any knowledge-based system, but has the particularity that we can have more than one rule fired simultaneously. In this situation, the question that arises is how to combine the actions of each of the heuristic rules fired. Implicitly is assumed a logic operation *OR* between rules, so that the knowledge base is read as *R1 OR R2*. As each rule is equivalent to an implication matrix, therefore, the knowledge base corresponds to a logic operation *OR* between two matrices, determined element by element. In general, we can say that, $R = vR_i$. Thus, the inference can be performed through the resulting matrix, *R*.

In the event that we have *n* inputs, i.e. the case where the antecedent of the heuristic rules will be consisting of *n* variables, the matrix, *R*, will be generalized taking the size *n+1*. Thus, considering that we have e_i inputs, with $i = 1, .., n$, the inference is performed through the generalized composition operation, which can be expressed as follows, $u = (e_1 \; x \; e_2 \; x ... x \; e_n) \circ R$. Thus, if there are *n* inputs, the inference continues to run through the operation of composition defined earlier, with only the need to take into account the dimensions concerned.

Fuzzy logic allows computers to work with inaccurate statements. The idea is to allow a gradual transition from true to false (or from yes to no, or from 0 to 1) according to circumstances. Thus, the controller of an air conditioner designed based on fuzzy logic may recognize the temperatures, hot and cold, of a room. The heuristic rules used in the design of such controllers, use statements less accurate than the rules formulated based on Boolean logic. An example of this type of rules is "*IF* the room temperature is hot *AND* is rising slightly *THEN* increase the air conditioner power slightly". As previously mentioned, the mathematical concept behind fuzzy logic is the fact that many classes, or sets, have ill-defined limits; the set of "hot temperatures" is an example of a set with imprecise boundaries.

Therefore, a block diagram of a feedback control loop using a fuzzy linguistic controller is shown in Figure 1. Thus, the fuzzy linguistic controller consists of a pre-processor block, a knowledge base, a defuzzifier and a postprocessor block. The pre-processor puts the input signal within a particular range of values and quantifies this value in terms of fuzzy sets defined in the universe of discourse of the corresponding variable. Using the information in the knowledge base, implication and inference operations are performed. The defuzzifier converts the fuzzy control signal into a classical scalar control signal. The postprocessor block is characterized by having an adjustable gain and often it is also used to implement control actions, as for instance, an integral control action over the controlled

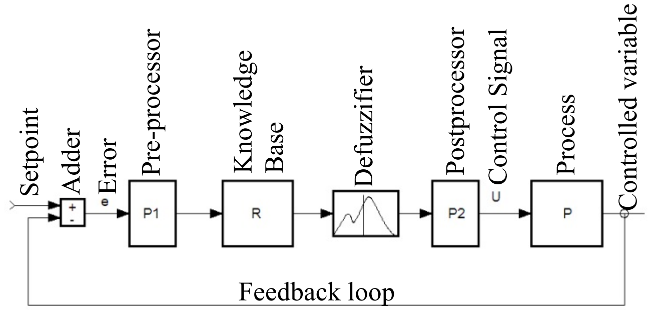

Figure 1. Fuzzy controller in a feedback control loop

variable. During the current studies the defuzzifier has been based on the most prevalent and physically appealing of all the defuzzification methods called Centroid Method, which has been implemented through the following equations [10],

$$u = \frac{\sum_i \mu(x_i)x_i}{\sum_i \mu(x_i)} \quad (3)$$

This method determines the coordinate x of a vertical line that divides an area into two equal halves. Equation (8) may be interpreted as a weighted average of all elements of a fuzzy set. In case we have a continuous representation of the membership functions, the sums of equation (8) are replaced by integrals. Many other methods of defuzzification can be found in the literature; studies have demonstrated that the methodologies based on the determination of maximum have good performance when dealing with fuzzy reasoning systems, while the methods that are based on determination of areas or distribution of values lead to better performance when it comes to fuzzy linguistic controllers [11].

III. CASE STUDY

A mixing process has been used as test bed of the rule base controller approach proposed in this paper. The layout of the mixing process is depicted in Figure 2. The hot water, at about 80 ^oC, is supplied from an electrically heated tank, while the cold water is supplied from the mains. Both streams enter tank 1 where mixing takes place. The contents of tank 1 pass to tank 2 and subsequently out to the pool tank from which they are recycled to the header tank. A number of hand valves can be seen in the mixing process shown in Figure 2. These hand valves are either kept fully open or fully closed during normal operation, as their function is simply to allow different systems configuration. When both tanks are used, as it has been considered during this research, han valves 1, 2, 3 and 5 are fully open and hand valve 4 is closed. The mixing process, although simple in operation, enables generic concepts to be developed. The simplicity of the process does not obscure the fundamental ideas that being studied. Measurement of the process variables level and temperature in both tanks is available and, hence, it is possible to control level and temperature in either tank. However, during the research work conducted with this process only the case of tank 2 being controlled has been considered. The hot inlet flow (HWCV) is used to control temperature while the cold inlet flow (CWCV) is used to control level. Since either hot inlet flow or cold inlet flow can affect both temperature and level, interaction exists between two control loops.

The dynamic model for the mixing process has been developed based on mass and heat balances relationships performed in tanks 1 and 2 respectively. Such a mathematical model has been used to simulate the process using the Labview computational platform run in a pc with the characteristics detailed in the next section. In short briefly, the dynamic model is presented below by the following equations,

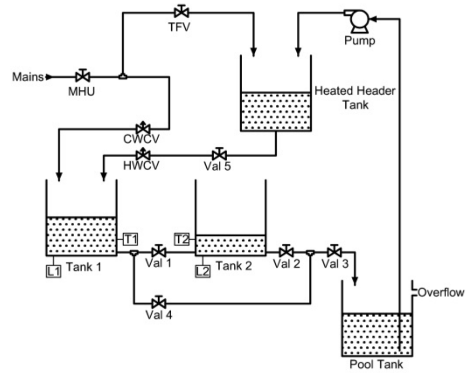

Figure 2. The mixing process layout.

$$A_1 \frac{dL_1}{dt} = Q_c + Q_h - Q_{o1} \quad (4)$$

$$A_2 \frac{dL_2}{dt} = Q_{o1} - Q_{o2} \quad (5)$$

$$A_1 L_1 \frac{dT_1}{dt} = Q_c(T_c - T_1) + Q_h(T_h - T_1) \quad (6)$$

$$A_2 L_2 \frac{dT_2}{dt} = Q_{01}(T_1 - T_2) \quad (7)$$

$$Q_{o1} = k_1\sqrt{L_1 - L_2} \quad (8)$$

$$Q_{o2} = k_2\sqrt{L_2} \quad (9)$$

Where the following notation is used: A_1, A_2 – cross-sectional area of tank 1 and 2 respectively (cm^2); L_1, L_2 – level in tank 1 and 2 respectively (cm); T_1, T_2 – temperature of water in tank 1 and 2 respectively (^oC); T_C – temperature of input cold water (^oC); T_h – temperature of input hot water (^oC); Q_c – input cold water flow rate (cm^3/s); Q_h – input hot water flow rate (cm^3/s); Q_{o1} – output water flow rate from tank 1 to tank 2 (cm^3/s); Q_{o2} – output water flow rate from tank 2 (cm^3/s); t – time (s); k_1, k_2 – constants characteristics of valves 1 and 2 respectively.

Moreover, during the simulation studies conducted with the dynamic model presented above the values used for the independent variables have been as follows: $A_1 = 290\ cm^2$; $A_2 = 150\ cm^2$; $T_h = 80\ ^oC$; $T_c = 20\ ^oC$; $k_1 = k_2 = 29\ cm^{5/2}/s$.

Furthermore, in the mixing process dynamic model presented above, the following assumptions have been made: difference between cold and hot water density is negligible; difference between cold and hot water specific heat is negligible; the mixing is perfect; water doesn't boil; heat transfer coefficients are constant; heat losses into the environment are negligible.

As previously mentioned during the current studies level and temperature in tank 2 have been controlled through feedback control loops aiming to cope with the coupling effects due to the interactions between the two control loops. In order to perform a results comparison two control strategies have been followed. Thus, the first experiences have been taken using the classical PID (proportional, integral, derivative) controllers; afterwards, the classical controllers have been replaced by fuzzy controllers based on *if-then* heuristic rules. The implementation details and the results comparison are described in the next section.

IV. Controllers and Results

As mentioned above the first experiments conducted with the mixing process include the control of level and temperature in tank 2 using a feedback loop control configuration being used PID controllers. The hot water inlet flow has been used to control the temperature while the cold water inlet flow has been used to control level. It was observed that all control actions were needed and, thus, the controllers' parameters are as follows: k – proportional gain; T_i – integral time (s); T_d – derivative time (s). The controllers' parameters have been tuned following the Ziegler-Nichols rules [12]. Therefore, the values for the controller parameters used in the level control loop are as follows: $k = 9.1$, $T_i = 150\ s$ and $T_d = 14\ s$. While the values for the controller parameters in the temperature control loop are as follows: $k = 3.7$, $T_i = 100\ s$ and $T_d = 5\ s$.

In order to access the controllers performance several step changes have been performed in the controlled variables setpoint and one of the results achieved is depicted in Figure 2, corresponding to a step change in the setpoint of the level control loop in tank 2. As can be observed in the graphs depicted in Figure 2 for the sake of simplicity the values of the process variables have been normalized and the values 1 in the y's axis correspond to 100 *cm* for level and 100 ^oC for temperature. The same occurs in the similar graphs. Having into consideration the values taken for the cold and hot water inlet streams the water temperature in tank 2 will be between 20 and 80 ^oC.

From the graphs depicted in Figure 3, it can be observed a big overshoot in the level control loop and a big interaction with the temperature control loop.

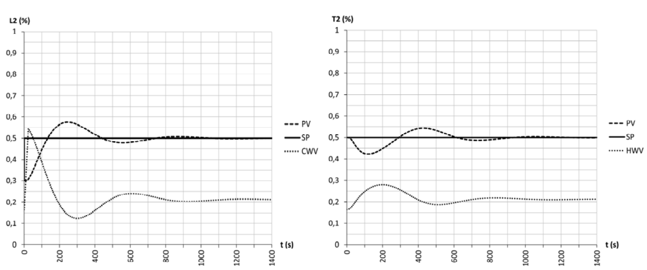

Figure 3. PID controllers' response to a setpoint change in the level control loop

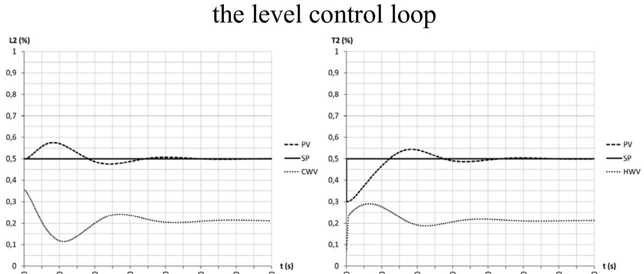

Figure 4. PID controllers' response to a setpoint change in the temperature control loop

TABLE I. Fuzzy Heuristic Rules

		de/dt		
		NEG	ZERO	POS
e	NEG	LNEG	SNEG	ZERO
	ZERO	SNEG	ZERO	SPOS
	POS	ZERO	SPOS	LPOS

In Figure 4 are depicted similar results to the ones depicted in the last figure, but now corresponding to a step change in the setpoint of the temperature control loop. The overshoot now is small but the interaction between control loops due to coupling effects is still big.

Therefore, aiming to overcome the PID controllers' poor performance shown in the last two figures, a fuzzy rule base controller based on *if-then* heuristic rules, as mentioned in previous sections, has been developed and implemented. The fuzzy heuristic rules have been developed aiming to mimic the behaviour of a classical controller with proportional and derivative control actions. Hence, the fuzzy linguistic controllers developed to replace the PID controllers have two input variables, the "error" and the "change in the error" that are fuzzified and, one fuzzy output variable, which is the control signal. Thus, the antecedent of the *if-then* heuristic rules consists of two fuzzy input variables linked by the *AND* connective. For the sake of simplicity all the fuzzy linguistic heuristic rules are presented in a table format as shown in Table I. The abbreviations *LNEG*, *SNEG*, *NEG*, *ZERO*, *POS*, *SPOS* and *LPOS* correspond respectively to the fuzzy sets (large negative, small negative, negative, zero, positive, small positive, large positive). As it can be observed from Table I, three fuzzy sets have been chosen to characterize the controller fuzzy input variables behaviour and five fuzzy sets have been used to characterize the behaviour of the controller fuzzy output variable. The corresponding membership functions are shown in Figure 5, 6 and 7, respectively for the fuzzy variables "error", "change in the error" and "control signal". All membership functions have been tuned following a trial and error procedure until has been achieved the controller desired performance.

Now replacing the PID controllers by fuzzy linguistic controllers in both control loops, level and temperature of

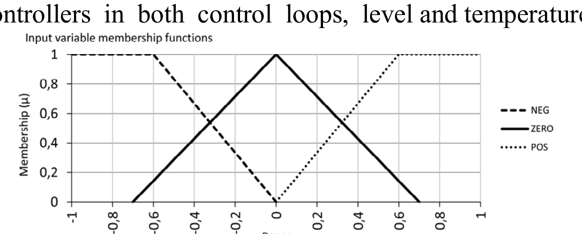

Figure 5. Membership functions of the fuzzy variable "error".

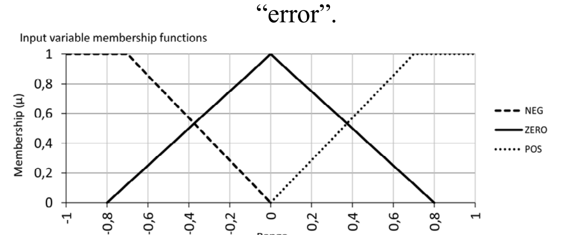

Figure 6. Membership functions of the fuzzy variable "change in the error".

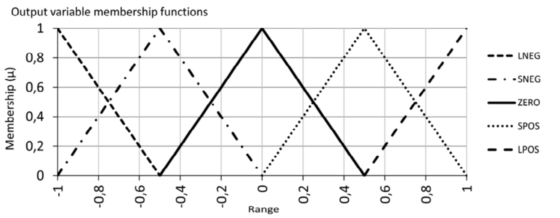

Figure 7. Membership functions of the fuzzy variable "control signal".

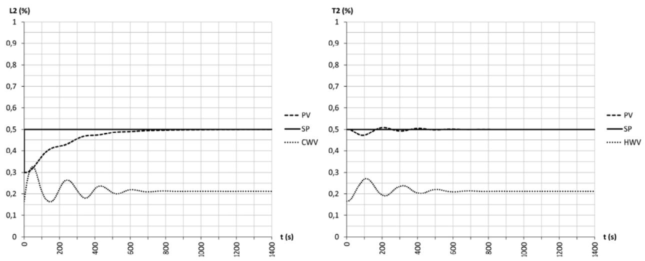

Figure 8. Fuzzy controllers' response to a setpoint change in the level control loop

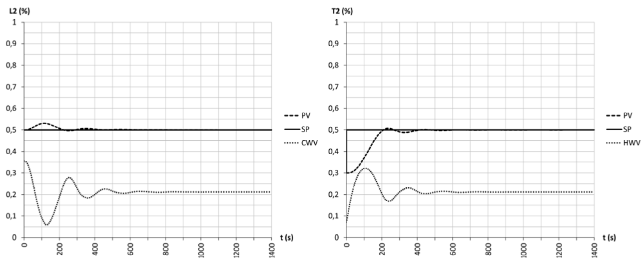

Figure 9. Fuzzy controllers' response to a setpoint change in the temperature control loop

tank 2, and making the same step changes in the setpoints of the controlled variables as shown in Figures 3 and 4, the corresponding controllers responses are depicted in Figures 8 and 9, respectively for a step change in the setpoint of the level control loop and in the setpoint of the temperature control loop. From Figures 8 and 9 can be observed that the performance of the fuzzy linguistic controllers is higher than the performance achieved using PID controllers. The response of the control loops to step changes in the setpoints of the controlled variables has low overshoot and a low settling time (time elapsed from the application of an ideal instantaneous step input to the time at which the controlled variable has entered and remained within a specified error band). Furthermore, it can be observed that the control loops interactions due to coupling effects are negligible.

All the simulations have been implemented using the Labview computational platform and run in an Intel Mobile Core 2 Duo, CPU T9300@2500GHz, DDR2 4096 Mbytes.

V. CONCLUSIONS

In order to cope with interactions between control loops due to coupling effects a fuzzy linguistic controller has been developed, implemented and tested. To access the fuzzy linguistic controllers' performance the control loops have also been implemented using the well-known PID controllers. As test bed of the proposed fuzzy linguistic controller a mixing process has been used. Successful results have been achieved for the performance of the fuzzy linguistic controllers when compared with the PID controllers' performance. Such results encourage the use of fuzzy linguistic controllers in processes with several control loops when interactions between them are observed. The fuzzy linguistic controllers almost complete eliminated the coupling effects between control loops.

Aiming to complete eliminate the coupling effects between control loops, further research will be conducted, namely in the field of fuzzy logic control, where a fuzzy linguistic controller will be designed based on the causal relationships between process variables and, the control actions being inferred from samples of both controlled and non-controlled process variables.

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