Application Note
#AN-00MX-012

Inclination Sensing Of Moving Vehicle

Introduction
This document describes a method to enhance the measurement of inclination in a moving vehicle. Road inclination sensing can be used to correct the fuel gauge reading while driving on steep inclined roads and in car navigation systems to enhance the accuracy of the coordinates obtained with the GPS (Global Positioning System).

Many methods are used to enhance the accuracy of the GPS coordinates when the signal is lost or degraded. The 'dead reckoning' method involves sensing the distance traveled with the odometer reading and the vehicle direction from the time the GPS signal is lost. Knowing the distance and direction this information can be matched against a road map database and used to infer the vehicle coordinates.

But even with dead reckoning the coordinates may not be sufficiently accurate, and in many instances the system can not distinguish between elevated roadways running parallel to ground level roadways. Another road condition difficult to discriminate by the navigation system is when the vehicle is moving in a multi-story parking structure. In these cases, system knowledge of the vehicle inclination (traveling up or down a ramp) can help enhance the accuracy of the estimated coordinates of the vehicle.

Measuring inclination of a stationary frame with thermal accelerometers is discussed in MEMSIC's application note #AN-00MX-007. In this app. note a method to enhance the measurement of the inclination (or pitch) of a moving vehicle is presented. The errors introduced by the accelerations and decelerations of the moving automobile are considered, and a way to quantify the inclination error due to the additional automobile accelerations is presented.

Error Using One Axis Accelerometer
When only a one axis accelerometer is used, the automobile accelerations or decelerations will be measured by the accelerometer, and the system would interpret these signals as changes in inclination.

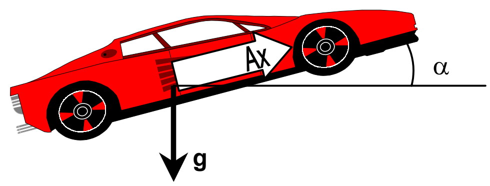

Figure 1.

The relationship between the accelerometer output and gravity is given by (reference Fig 1):

$$Ax = g \cdot \sin(\alpha)$$

Hence, the inclination angle is defined as:

$$\alpha = \sin^{-1}(Ax/g)$$

From the above equation it is obvious that during vehicle accelerations and decelerations Ax will change and an error is introduced in the inclination measurement.

For example, a vehicle accelerating on a level freeway ramp from 10MPH to 55MPH in 10 seconds will experience an acceleration of 0.2g. The measuring system would interpret the acceleration as an inclination of almost 12°arc.

An alternative way to measure the inclination angle is to mount the single axis accelerometer so that its sensing axis is perpendicular to the vehicle accelerations.

In this orientation the accelerometer would only measure the effect of gravity, and the inclination angle is given by (reference Fig.2):

$$\alpha = \cos^{-1}(Ax/g)$$

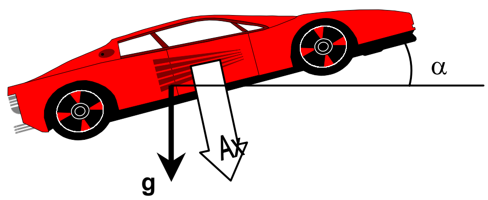

Figure 2.

Unfortunately, by the nature of the cosine function the typical road inclination angles represent very small changes in acceleration. For example, a 10°arc slope is equivalent to an acceleration of 0.985 g. The output change from horizontal is 0.015 g, and this is a very small signal relative to the accuracy of most MEMS accelerometers available today. In this orientation the inclination measurement is simple, but not sufficiently accurate.

Improvement With Dual Axis Accelerometer

Using a dual axis accelerometers, the automobile accelerations can be measured separately from the constant acceleration of gravity. The automobile accelerations and gravity occur along two axis that lie on a vertical plane. So by placing the dual axis accelerometer sensing axes on the vertical plane and applying some trigonometric relations, the inclination angle measurement is enhanced.

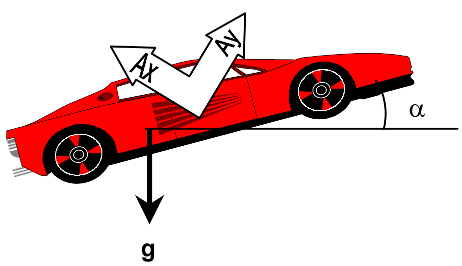

Figure 3.

Given that the accelerometer sensing axes are mounted at 45°arc from the longitudinal axis of the automobile, the inclination angle, independent of automobile accelerations is given by (reference Appendix derivation):

$$\alpha = \cos^{-1} (0.707 \cdot [Ax+Ay]/g)$$

One can note from the above relation that accelerations along the automobile longitudinal axis will be of similar magnitude but opposite in polarity, thereby keeping the (Ax + Ay) term relatively constant. For example, when the automobile accelerates forward, Ax increases while Ay decreases.

System Implementation

Since it may not be convenient or possible to assure that the accelerometer is mounted at exactly 45°arc from the longitudinal axis of the vehicle, an initial calibration can be performed to measure the actual mounting angle while the automobile is parked on a horizontal surface. The mounting angle can be calculated by:

$$\beta = \tan^{-1} [Ay / Ax]$$

Once the angle is known, the general form of the inclination angle equation derived in the Appendix can be used:

$$\alpha = \cos^{-1} ([\sin(\beta) \cdot Ax + \cos(\beta) \cdot Ay]/g)$$

An alternative method to the initial calibration mentioned above, is to have the system run a continuous average of Ax and Ay. Most roads are leveled so after a short time of operation, the system will "learn" the mounting angle. The longer the system is operated the more accurate it will perform.

Conclusion

MEMSIC accelerometers are low cost, high reliability and rugged instruments that can accurately measure the inclination or pitch of a moving automobile. Thermal accelerometers have no solid moving parts, so they do not display poor reliability due to stiction and particle contamination that are common to capacitive type MEMS accelerometers.

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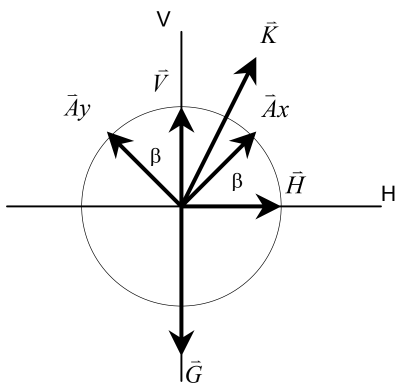
Appendix

Derivation of Inclination Angle From Dual Axis Accelerometer Output

Nomenclature:
$\vec{A}x$: accelerometer first sense axis unit vector
$\vec{A}y$: accelerometer second sense axis unit vector
$\vec{H}$: horizontal axis unit vector
$\vec{V}$: vertical axis unit vector
$\vec{G}$: gravity vector
$\vec{K}$: some vector
β: angle between accelerometer and vehicle
ax: first accelerometer output
ay: second accelerometer output
h: horizontal acceleration component
v: vertical acceleration component



$\vec{H} = \cos\beta \cdot \vec{A}x - \sin\beta \cdot \vec{A}y$ ………(1)

$\vec{V} = \cos\beta \cdot \vec{A}x + \sin\beta \cdot \vec{A}y$ ………(2)

$\vec{G} = g \cdot \vec{V}$ …………………………(3)

$\vec{K} = ax \cdot \vec{A}x + ay \cdot \vec{A}y$ …………..(4)

$\vec{K} = h \cdot \vec{H} + v \cdot \vec{V}$ ……………..…..(5)

From (1) (2) (3) (4)

$\vec{K} = h \cdot (\cos\beta \cdot \vec{A}x - \sin\beta \cdot \vec{A}y) + v \cdot (\sin\beta \cdot \vec{A}x + \cos\beta \cdot \vec{A}y)$

$\vec{K} = (h \cdot \cos\beta \cdot + v \cdot \sin\beta) \cdot \vec{A}x + (-h \cdot \sin\beta + v \cdot \cos\beta) \cdot \vec{A}y$ (6)

From (4) (6)

$ax = h \cdot \cos\beta + v \cdot \sin\beta$ …………..(7)

$ay = -h \cdot \sin\beta + v \cdot \cos\beta$ …………..(8)

From (7)

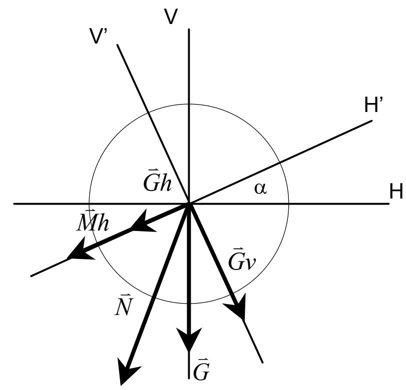
$h = (ax - v \cdot \sin\beta) / \cos\beta$ …………..(9)

From (8) (9)

$ay = -(ax - v \cdot \sin\beta) \cdot \sin\beta / \cos\beta + v \cdot \cos\beta$

Solve for v,

$v = ax \cdot \sin\beta + ay \cdot \cos\beta$



With the car on the slope with angle α, the accelerometer outputs are also rotated by the angle α.

$\vec{M}h$: acceleration of the car
$\vec{G}h$: accel. horizontal component from gravity
$\vec{G}v$: accel. vertical component from gravity
$\vec{N}$: combined vector of acceleration and gravity
α: vehicle inclination angle

$\cos\alpha = |\vec{G}v| / |\vec{G}|$

$\cos\alpha = (ax \cdot \sin\beta + ay \cdot \cos\beta) / g$

The inclination angle is given by

$$\alpha = \cos^{-1}[(ax \cdot \sin\beta + ay \cdot \cos\beta) / g]$$